Transformation of the L*A*B* Color Space to Obtain a More Uniform Chromaticity Diagram

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Abstract

The color matching ellipsoid theory has been extensively used to improve the relationship between the human perception and numerical representation of color differences evaluation. This theory can be described thanks to logarithmic functions which approximate the orientation, the position and the size of ellipses. By carrying out these observations to the study of the a*b* chromaticity plane we have obtain a partition of this plane. This partition separates each area for which the distribution of ellipses can be locally approximated by the same logarithmic function. Then we can interpolate the orientation and the dimension of ellipses for each position of each area

This logarithmic function points out that the perception of colors is non-uniform at once "locally" for each area considered separately and "globally" for the set of areas. Inversely, we can claim that if we take the "inverse function" of this logarithmic function we can transform the distribution of ellipses in a convenient form in order to obtain an uniform distribution of circles. This process can be applied to each area of the chromatic plane and can be scaled to compute a distribution composed of equal-sized circles. Then, this transformation allows to set a new color space which is more uniform than the L*a*b* color space: we will call it the L°a°b° color space.

Keyword: uniform color space, color matching ellipses, color difference evaluation

1. Introduction

The L*a*b* color space has been defined by the CIE[1] (Commission International de l'Eclairage) in 1976. This color space is actually one of the most satisfactory from the standpoint of color measurement and color matching.[2,3] Nevertheless, this color space is not completely consistent with the human perception of color differences. Several works have attempt to supply a more uniform color space to avoid this problem. Unfortunately it has been mathematically proved that it is not possible to obtain an uniform color space.[3] The purpose of our investigation is then to define at best a color space approximately uniform.

The first part of this article surveys some of the most important results obtained by analyzing the visual sensitivities to color differences. More complete surveys had been carried out by Wright,[4] MacAdam,[5] Wyszecki and Stiles.[6] The purpose of our investigation is only to show that the ellipsoid model established by Brown and MacAdam is the most relevant one to describe and to analyze the deviation of color measurement or color matching from one color point to an another color point. The second part analyzes more finely the standard deviation of ellipses in several directions representing certain visual sensitivities. A logarithmic transformation is then proposed to equalize the length of each projective ellipse in each of these directions. This process is then repeated until all ellipses are transformed into equal-sized circles. The last part of this article gives the first results that we have obtained.

Several works concern the visual sensitivities of color differences. Among these works we can consider that the study of Wright[4] is undoubtly the most interesting one due to the associated experimental observations. In his work Wright investigated the measurement of small color steps for various lines across the CIE color chart. The results have been illustrated graphically by a serie of short lines of the appropriate length drawn on the color diagram (Figure 1).

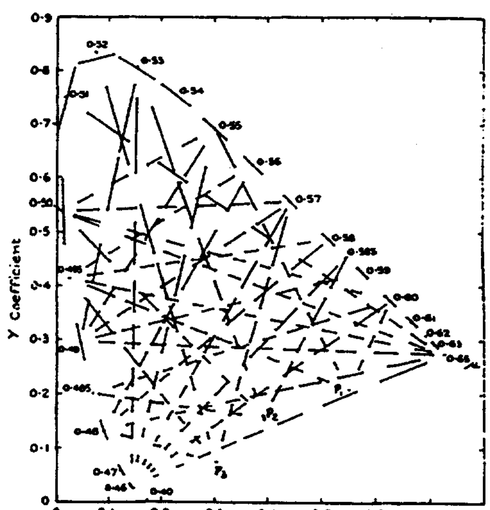

Figure 1. Color steps shown in the CIE chart by their actual length Δl, at various points along 35 lines, according to Wright.[4]

The previous results show the very wide size variation of steps in different parts of the color diagram. They prove that to a given distance, at any point and in any direction in the color chart, does not correspond an equal sensory color difference. These results have been illustrated in the 1931 CIE XYZ standard colorimetric system but can be easily transposed to the 1976 CIE L*a*b* color space.[2] The first color study concerned the analysis of the variation of sensitivity to small color differences in the 1931 CIE XYZ color space. Then, this study has been extended to other color spaces to characterize their variation to color differences measurement. As an example the L*a*b* color space is more uniform than the 1931 CIE XYZ standard colorimetric system.[2]

2. Standard Deviation of Chromaticity Sensitivity to Color Differences

Another approach has been proposed by MacAdam[5] to analyze the variation of sensitivity to small color differences. This approach is more relevant than the previous one because it is less sensitive to the subjectivity of experimental observations.[4,5] Even if the previous study is less significant in terms of differences of color sensitivities (due to the fact that it has been done with only a single observer and only for a short number of color points), it is more elaborated since the author considered for each color point not only one direction but at least 8 directions. Moreover in this approach, measurements of color matching are directly related to the corresponding just noticeable difference of colors. In this article, we only discuss about chromaticity differences, because the main results which have been published are essentially based on chromaticity analysis. For more information on the general case of color discrimination, we refer the reader to the Brown and MacAdam's investigation.[7] Their approach is based on studies combined chromaticity and luminance differences

For each color point and for each direction, two opposite observation points from the center point are thus determined corresponding to the limits of color matching. For each color point, an ellipse is then drawn through these opposite points. This ellipse represents the noticeability of chromaticity variations in all directions from the chromaticity indicated at the center of the ellipse (Figure 2).

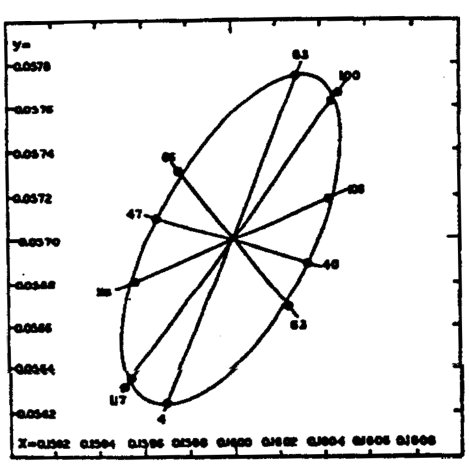

Figure 2. Standard deviations of chromaticity from point (x = 0.160, y = 0.057), for the observer PGN, according MacAdam.[5]

Even if these ellipses are based on extrapolations of observed data, it seems that experimentally they are correlated significantly to the observations. All these ellipses can be represented to the same scale on a composite diagram as in Figure 3.

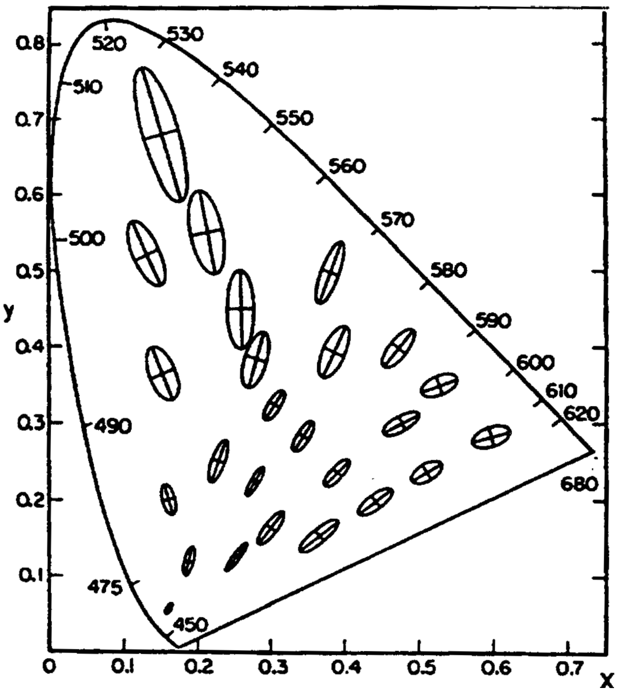

Figure 3. Standard deviations of chromaticity from indicated standards in the 1931 standard chromaticity diagram, for the observer PGN, according to MacAdam.[7] Every ellipse is drawn ten times its correct size with relation to the coordinate scale of the chromaticity diagram. The centers of the ellipses are placed at their proper locations in the chromaticity diagram.

Consequently, each ellipse can be described by three values G_{11}, G_{12}, G_{22} according to the equation:

$$G_{11}\,(x, y)\,(\Delta x)^2 + 2\,G_{12}(x, y)\,(\Delta x)(\Delta y) + G_{22}\,(x, y)(\Delta y)^2 = 1 \qquad (1)$$

where Δx (resp. Δy) represents the standard deviation from the center point toward the intersection point between the ellipse and the direction given by the x axis (resp. the y axis).

By interpolation, we can compute at each point (x,y) of the chromaticity plane the coefficients $G_{11}(x, y)$,$G_{12}(x, y)$,$G_{22}(x, y)$.[7] Consequently, at each point of the chromaticity plane corresponds an ellipse which represents the standard deviation of chromaticity sensitivity at this point. As an example see Figures 4 and 5.

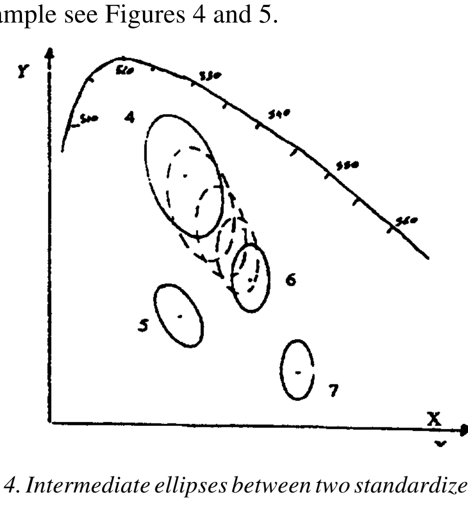

Figure 4. Intermediate ellipses between two standardized ellipses

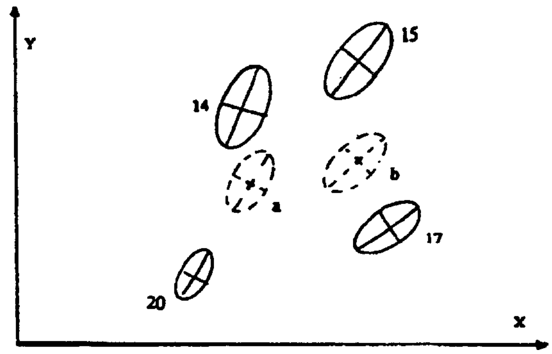

Figure 5. Two intermediate ellipses between several standardized ellipses.

It has been shown that these interpolated coefficients corroborate significantly the coefficients which represent the original experimental data. It seems then that the 25 ellipses given by MacAdam are enough representative to describe entirely the chromaticity plane in terms of standard deviation of chromaticity sensitivity. Several of our developments are based on this characteristic because it simplifies significantly the computations.

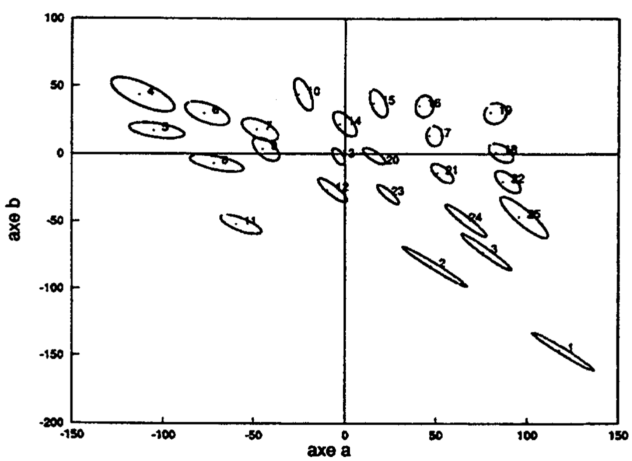

*Figure 6. Standard deviations of chromaticity from indicated standards in the 1976 a*b* chromaticity diagram, at a constant luminance L*=50.*

MacAdam has firstly defined his elliptic model to analyze the variation of sensitivity to small color differences in the 1931 CIE XYZ color space. Identically to the Wright's descriptor, the MacAdam's descriptor can be transposed to the L*a*b* color space. We can then show once again that this color space is more uniform than the previous one (Figure 6).

Nevertheless, this color space can not be considered as an ideal color space because colors of equal chromaticness are not located on circles with equal perceived distances corresponding to equal geometric distances.

3. Hypotheses on the Distribution of Ellipses

Several criteria can be used to describe the distribution of colors of equal chromaticness. Among these criteria we propose to consider the length of projection of each ellipse on each axis of the (a*, b*) chromaticity diagram. Let $\Delta(a_i^*)$ and $\Delta(b_i^*)$ be the lengths of projection of the ellipse centered on the color point (a_i^*, b_i^*), respectively on the axis a* and on the axis b* (Figure 7).

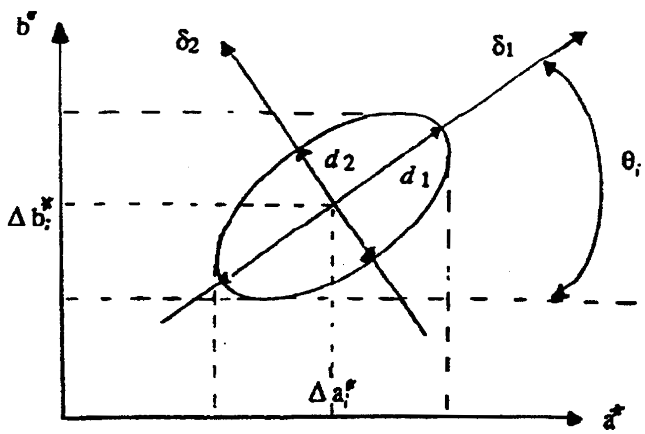

Figure 7. Lengths of projection of an ellipse on the axes a and b*.*

Then from Table 1, we can quantify the standard deviation of chromaticity sensitivity to color difference.

Moreover, from Figure 8 we can see that the size and the direction of the ellipses seem to be correlated with their position, and that from one position to another one the size and the direction of these ellipses seem to be related to the

Table 1. Lengths of projection of each ellipse on the axes a* and b*. These ellipses correspond to the cross sections of the 25 ellipsoids defined by MacAdam, transposed in the (a*, b*) chromaticity diagram, at a constant luminance L* = 50.

Ellipse No i	Δai*	Δbi*	Ellipses No i	Δai*	Δbi*	Ellipses No i	Δai*	Δbi*
1	3.3757	2.8073	9	1.4717	1.7452	17	0.8781	1.4750
2	3.5549	2.9773	10	1.0134	2.3747	18	1.3476	1.3987
3	2.7220	2.6884	11	2.3031	1.4343	19	1.2177	1.5284
4	3.4438	2.5165	12	1.5753	1.7622	20	1.2230	1.1891
5	3.0018	1.1715	13	0.6879	1.2124	21	1.2226	1.3881
6	2.4659	1.7155	14	1.2910	1.8208	22	1.3742	1.6373
7	2.0030	1.6718	15	0.9780	2.0323	23	1.2168	1.4576
8	3.0000	1.2460	16	1.0019	1.5882	24	2.2510	2.4283
						25	2.5665	3.0740

same phenomenon. As an example, we can see in this Figure that the ellipses 4, 6, 7, 9 and 13 seem to be distributed along the same direction and that the more they are distant from the center of the (a*, b*) chromaticity plane the more their size grow.

This aspect can be also studied more finely thanks to the diagrams of lengths of projection of standardized ellipses (Figures 9 and 10). As an example we can see on the Figure 11 that the ellipses 4, 6, 7, 9 and 13 have their lengths of projection on the axis a* which are approximately distributed according to a linear law.

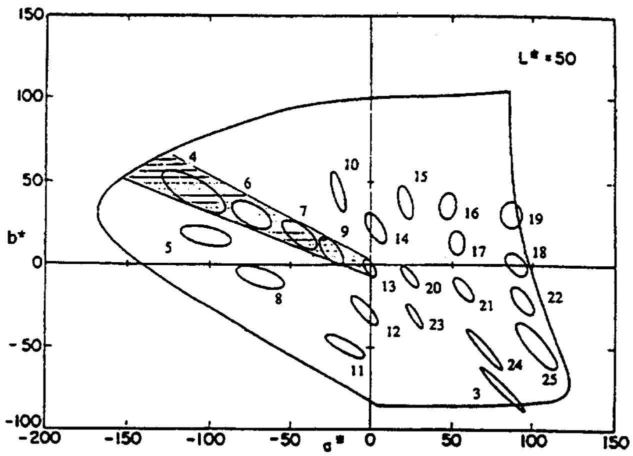

Figure 8. Section of the (a,b*) chromaticity diagram where the ellipses distribution follows the same behavior law (same direction, size growing). This law is only defined by the standardized ellipses 4, 6, 7, 9 and 13, even if actually we can consider that all intermediate ellipses participate indirectly to its definition.*

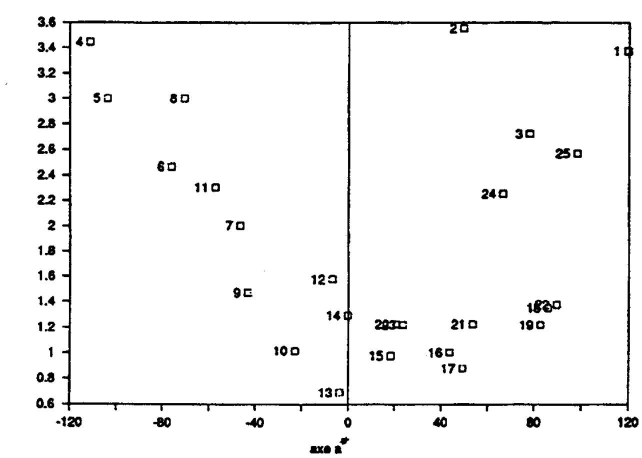

*Figure 9. Lengths of projection of each standardized ellipse on the axis a**

Two remarks need to be formulated at this stage of our investigation. Firstly, it seems that this notion of specific direction could be justified in terms of chromaticity sensitivity, in particular when we refer to the theory of confusion points (Figure 12). Secondly, even if the chromaticity diagram is made up of two perceptually distinct dimensions which can be analyzed separately, it is essential to keep in mind that these dimensions could be dependent upon direction or location regardless the criterion under study.[8] In our case of study, we analyze the distribution of ellipses according their lengths of projection. Each axis is then studied independently of the other. Nevertheless, we can underline that the length of each projection illustrates in the same time the size and the inclination of the corresponding ellipse (see Figure 7). Consequently, each length of projection is directly proportional to the other corresponding length of projection. Thus, even if a separate scale has been constructed for each of the underlying dimensions, it still exist data which permit to find again the initial dependence of ellipses distribution upon location or direction along the chromaticity plane. Thanks to the geometric shape of ellipses, which can be seen as isosimilarity contours, it is possible to construct a spatial representation from the two unidimensional scales analogous to the bi-dimensional diagram.[8]

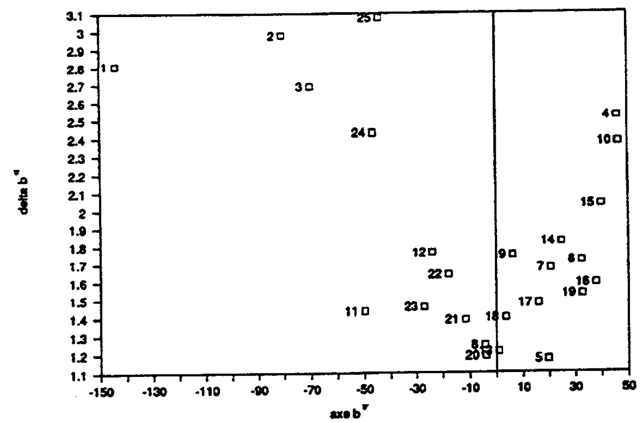

*Figure 10. Lengths of projection of each standardized ellipse on the axis b**

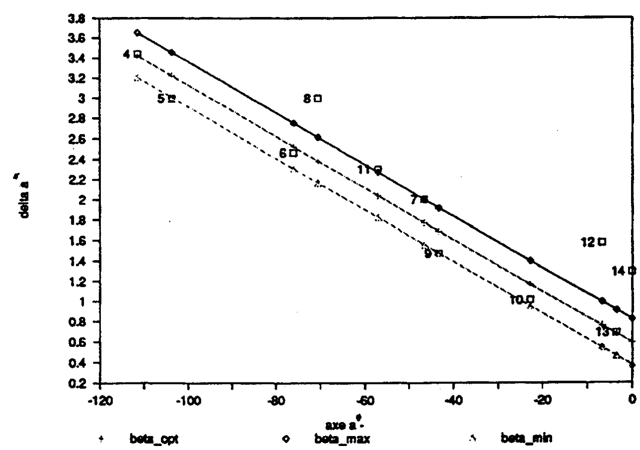

Figure 11. Length of projection $\Delta(a_i*)$ on the axis a of each standardized ellipse centered on the color point (a_i*, b_i*). Linear adjustment, according to the principle of least-squared error, of ellipses 4,6,7,9 and 13. Other ellipses are not concerned by this adjustment because they are centered in an another section of the chromaticity diagram.*

4. Developments Step by Step of the Proposed Process

Let us now come back to each projection diagram. Considering the distribution of data (a_i*,$\Delta(a_i*)$), we have observed that for each section of the chromaticity diagram the length of projection on the axis a* of all ellipses clustered in this section are approximately distributed according to a linear law. Consequently, a linear adjustment can be defined according to the principle of the least-squared error to characterize the distribution of data $(a_i^*, \Delta(a_i^*))$. A same observation can be done for the distribution of data $(b_i^*, \Delta(b_i^*))$.

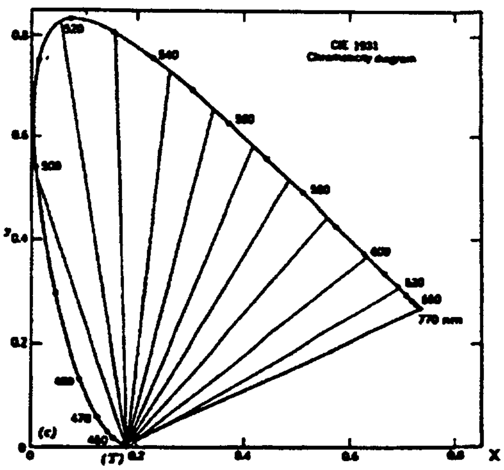

Figure 12. Lines of constant dichromatic chromaticity and confusion point (1) drawn in the 1931 CIE (x,y) chromaticity diagram, for a tritanope, according Judd and Wyszecki.[6]

Let $\Delta_0(a^*)$ be the ideal linear adjustment and $\Delta(a^*)$ be the real linear adjustment defined from the experimental data.

If $$\Delta_0(a^*) = \alpha\, a^* + \beta \qquad (2)$$

Then $$\Delta(a^*) = \Delta_0(a^*) + \varepsilon(a^*) = (\alpha\, a^* + \beta) + \varepsilon(a^*) \qquad (3)$$

where $\varepsilon(a^*)$ represents the error of fitting resulting from the considered linear adjustment.

Then, $\varepsilon(a^*)$ can be computed by considering the distribution of $\varepsilon(a_i^*)$ given by:

$$\varepsilon(a_i^*) = \Delta(a_i^*) - (\alpha\, a_i^* + \beta) \qquad (4)$$

and thanks to the least-squared average error ε resulting from this linear adjustment.

At this stage of our investigation we can notice that for some sections of the chromaticity diagram the size of the ellipses grow proportionally to their distance to the center of the chromaticity plane. Inversely, for other sections, the size of the ellipses decrease proportionally to this distance. Likewise, the inclination of these ellipses is directly linked to their position in the chromaticity diagram. These two aspects also appear when studying the lengths of projection on each axis of the chromaticity diagram. Consequently it will be very interesting to find a monotonic function for each section of the chromaticity diagram that will stretch or shrink each ellipse according to the one-dimensional scale which represents their linear adjustment. Then two color points separated by a fixed distance on the resulting transformed scale will always represent equally chromaticity sensitivity to color differences, regardless of the location of the pair on the original scale.

For each section of the chromaticity diagram, we propose to define a function f_0 that transforms separately each axis to adjust lengths of projection to equal standard lengths.

Suppose that a* is the axis under study and that a° is its transformation by the function f_0 such as:

$$a° = f_0(a^*) \qquad (5)$$

By taking into account the previous linear law defined on the lengths of projection $\Delta(a^*)$, we can define a function f that makes uniform to 1 each length of projection. Thus, f needs to verify:

$$f(a^* + \Delta(a^*)/2) - f(a^* - \Delta(a^*)/2) = 1 \qquad (6)$$

This can be illustrated by the Figure 13.

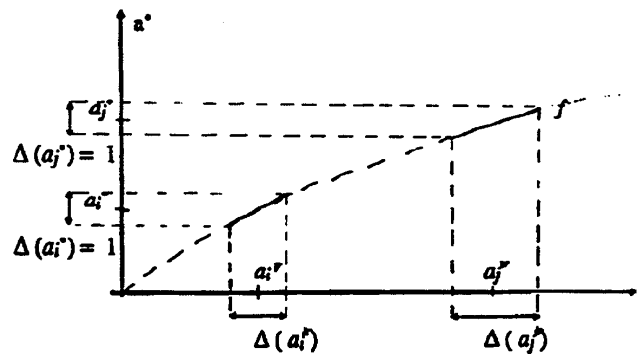

Figure 13. Illustration of a function f which makes uniform to 1 each length of projection $\Delta(a_i^*)$ of each ellipse (centered on the point (a_i^*, b_i^*)), according to the axis a. a° represents the new axis obtained by this transformation ($\Delta(a°) = 1$)*

Then, by reasoning to the first order relatively to $\Delta(a^*)$ we have:

$$\Delta(a^*)\, f'(a^*) = 1. \qquad (7)$$

This relation yields a general definition for the function:

$$f(a^*) = \int 1./\Delta(a^*)\, da^* \qquad (8)$$

This definition can be applied to the particular case of the linear adjustment of the length of projection, thus f_0 can be defined by :

$$f_0(a^*) = (1./\alpha)\, Log(\alpha\, a^* + \beta) + \gamma \qquad (9)$$

To take into account the errors of adjustment resulting from the considered linear law, we propose to define f according to the following formula:

$$f(a^*) = f_0(a^*) + \xi(a^*) = (1./\alpha)\, Log(\alpha\, a^* + \beta) + \zeta(a^*) \qquad (10)$$

This function makes uniform to 1 each length of projection of each ellipse according to the linear law which modelizes their distribution. Consequently, the error of adjustment resulting from the linear law appears naturally in the definition of f through the term $\varepsilon(a^*)$.

Thus, $\zeta(a^*)$ can be defined by:

$$\zeta(a^*) = -\int \varepsilon(a^*)/\, \Delta_0(a^*)^2\, da^* \qquad (11)$$

This transformation that makes uniform to 1 each length of projection has been applied to each section of the chromaticity diagram. It implies firstly to partition the chromaticity plane in several sections for which the ellipses distribution can be adjusted by a linear law without increasing too much the number of sections which can be defined. It implies also to compute the coefficients α and β without increasing too much the errors of adjustment. In this way, we have empirically established a compromise between the number of sections relatively to the 25 standardized ellipses and the amplitude of the errors of adjustment relatively to the uniformisation to 1.

In order to conserve the continuous feature of the chromaticity diagram, we have constructed new sections which are defined as intermediate areas between each pair of consecutive sections. Then, we have provided these intermediate areas with a function *f* for which the coefficients have been computed by interpolation of the different coefficients of the underlying function *f*. Thanks to this approach we have also conserved the coherence of the chromaticity diagram in terms of surface plane where the new axes are straight. This aspect is essential because it allows the use of the Euclidean geometry with the least-squared distance. Inversely, if we had had a curved diagram it would be necessary to use the Riemannian geometry with the geodesic distance.[9,10,3]

The obtained results confirm our hypothesis. Thus, the new lengths of projection are more relevant than the previous one (Table 2).

It still exist a small fluctuation around the values of lengths of projection but this one is lesser than in the (a*, b*) chromaticity diagram. This small fluctuation is inherent in our approach, it comes from the errors of adjustment linked to the use of the linear law.

This constitutes the first part of our transformation, of our new chromaticity diagram. It decreases significantly the standard deviation of chromaticity sensitivity to color difference over the chromaticity diagram, but it does not resolve entirely the problem of elliptic distribution. To illustrate our purpose consider two cases of study (Figure 14).

According to the inclination of the ellipses in the (a*, b*) chromaticity diagram, we have to face to two borderline cases. Either the direction of the ellipses is parallel to one axis: the initial dependence upon axes is minimal, consequently our process transforms effectively the ellipse into a circle. Either this direction is oblique, the initial dependence upon axes is then maximal, consequently our process even if it makes uniform the lengths of projection can not transform the ellipse into a circle. This problem is in fact quite easy to solve. Indeed, by knowing the inclination of the ellipse under study, it is easy to transform the system of axes to obtain a new system for which the axes are parallel to this ellipse. By this way, we are systematically dealing with the first case of study.

Nevertheless, rather than changing systematically the system of axes according to the inclination of each ellipse, that is excessively costly and aberrant in terms of computation considering that at each color point of the chromaticity diagram corresponds an ellipse, we can used once again the different linear laws previously defined on each section in order to establish sets of ellipses for which inclinations are identical.

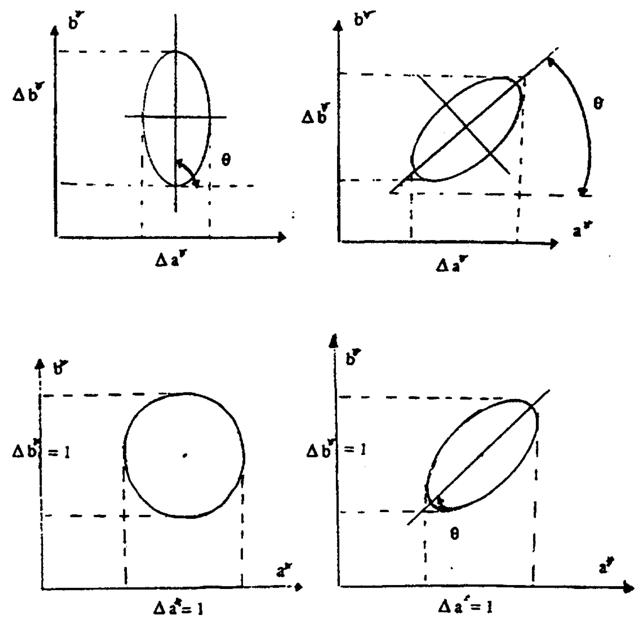

Figure 14. Borderline cases to make uniform to 1 the lengths of projection of an ellipse separately on each axis.

Table 2. Lengths of projection of each ellipse on the axes a° and b°. These ellipses correspond to the cross sections of the 25 ellipsoids defined by MacAdam, transposed in new (a°, b°) chromaticity diagram that we have defined, at a constant luminance L* = 50.

Ellipse No i	Δai*	Δbi*	Ellipses No i	Δai*	Δbi*	Ellipses No i	Δai*	Δbi*
1	0.9999	1.0001	9	0.7114	1.1896	17	0.7252	1.0054
2	1.0219	1.0000	10	1.0002	0.9547	18	1.0280	0.9534
3	0.9234	1.0001	11	1.0087	1.0002	19	0.9351	1.0004
4	1.1034	1.0251	12	0.8700	0.9999	20	1.0796	0.8105
5	0.9999	0.7986	13	0.6452	0.8265	21	0.9999	0.9462
6	0.9577	1.1401	14	1.2003	1.2411	22	1.0401	1.1161
7	0.9451	1.1396	15	0.8684	1.0004	23	1.0666	0.7629
8	0.9671	0.8494	16	0.8379	0.8420	24	1.0001	0.8670
						25	0.8506	1.1403

To summarize our approach, we have in a first time proposed to make uniform to 1 each length of projection of each ellipse, and in a second time we have proposed a complementary process which can moreover transform the resulting ellipses into circles. These two processes can be combined in an iterative process which alternates successively each of these two processes until a chromaticity diagram is obtain where equal-sized circles takes place instead of the previous ellipses. To illustrate this process we can take an example which involves two consecutive sections (Figure 15).

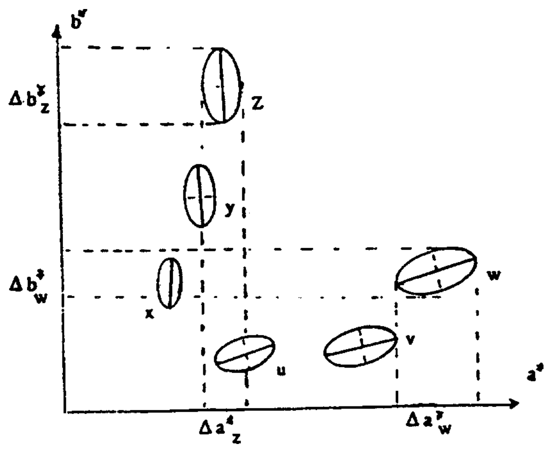

Figure 15. Example of an ellipses distributor involving two consecutive sections in the (a, b*) chromaticity diagram. Two sections can be constructed: the first one is organized around the standardized ellipses x,y,z and the second one is organized around the standardized ellipses u,v,w.*

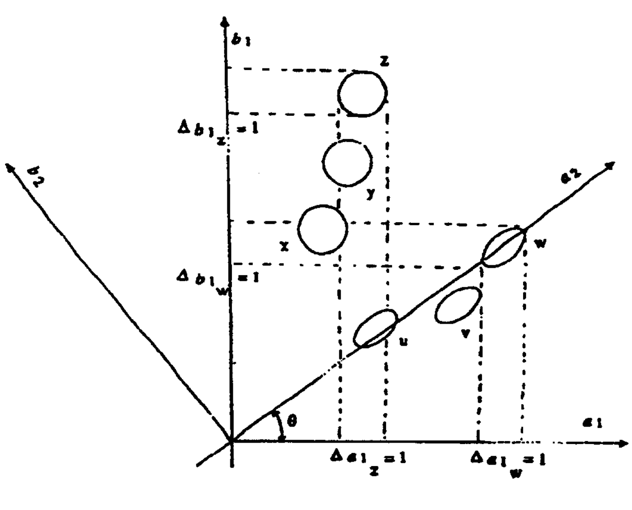

Figure 16. First and second step of the process. The first step makes uniform each length of projection through the transformation (a, b*) -> (a_1, b_1). The second step modifies the orientation of the system of axes through the transformation*

(a_1, b_1) -> (a_2, b_2).

Two remarks can be done: the ellipses x,y,z which are oriented in the direction of the axis b are now transformed into circles, and the ellipses u,v,w are now oriented in the direction of the new axis a_2.*

The first section is organized from the standardized ellipses x,y,z, the second one is organized from the standardized ellipses u,v,w. The first step of this process makes uniform each length of projection of each ellipse on each axe according to the linear law associated to these ellipses.

Then, in the second part of the process, section by section, from the first section to the last one, in the anticlockwise direction, the current system of axes is transformed to obtain a new system of axes for which the inclination of all ellipses involved by this section are approximately in the direction of this new system.

Thus, in our example a new system of axes (a_2, b_2) oriented according to the ellipses u,v,w has been defined from the system of axes (a_1, b_1) (Figure 16).

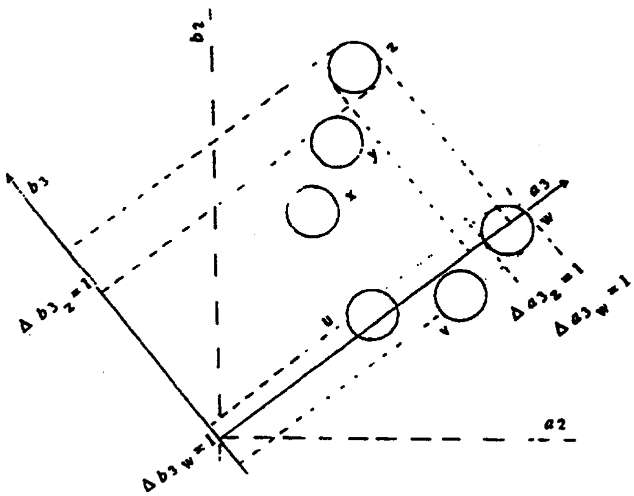

Figure 17. Third step of the process. It makes uniform each length of projection according to the a_2 and b_2 directions through the transformation (a_2, b_2) -> (a_3, b_3). The equal-sized circles x,y,z are not concerned by this transformation because their lengths of projection are already uniformized to 1. Inversely, the ellipse u,v,w are now transformed in circles. Instead of ellipses we have everywhere equal-sized circles: our objective is consequently reached.

The third step of our process consists to repeat the first step from the new system of axes, i.e. to make uniform the lengths of projection of each ellipse according new directions. At this stage of the process, we can remark that all the ellipses which have been already transformed into circles are not modified by this process, and that all the ellipses which are linked to the new system of axes are henceforth transformed into circles (Figure 17). Consequently, this process converges progressively toward an uniformization of all ellipses into approximately equal-sized circles. Once again a small fluctuation on the size of circles appears due to the linear adjustment of the ellipses distribution used to compute the orientation of each new system of axes. Nevertheless, the obtained results are enough relevant to claim that our attempts are satisfied and that the linear model even if it is empirical and not highly developed can be used with success to provide a more precise representation of color difference in terms of standard deviation of chromaticity sensitivity.

Figure 18 : Original color image
(256*256 picture elements)

Figure 19 : Image quantized according to the variance-based algorithm[15] applied to the RGB color space.

Figure 20 : Image quantized according to the variance-based algorithm[15] applied to the L*a*b* color space.

Figure 21 : Image quantized according to the variance-based algorithm[15] applied to the L°a°b° color space.

5. Discussion

At this stage of the process, it seems interesting to make two observations. Firstly, most of the computations to transform equal perceived distances to equal geometric distances are made in the first step of the process. The other steps participate only to improve the circular aspect of some distributions of ellipses. That is the reason why we have only given results linked to the first step. Secondly, there is a lack of data in some parts of the chromaticity diagram, especially in the blue part where only three ellipses (labeled 1,2,3) are available.[11] Consequently, it is very hard to establish a robust adjustment of these data whatever the monotonic function that we can used.

The same study can be executed at different levels of luminance. Thus in this article, we have exhibited the case of cross sections of the L*a*b* color space at a constant luminance level $L^* = 50$. We would develop the same kind of transformations for the levels $L^* = 30$ and $L^* = 70$ according to the data given by MacAdam and Pointer.[2] Then by interpolation we have defined all the intermediate levels and the external levels, in taking into account the Weber's law. This process has given results which corroborate significantly experimental data.

In all our investigation we have stressed on the principles and the different steps of our process of uniformization without detailed all intermediate computations. We think that what is really interesting in this approach is the possibility to define mathematical transformations based on physiological characteristics which are relatively simple to construct and which permits to adjust perceived distances to equal geometric distances in color difference perception. The proposed transformation is useful in practice even if it seems a little complicated. It consists in a set of logarithmic transformations defined according to a set of sections. All the sections and all the coefficients of the logarithmic transformations have been previously computed and tabulated. Consequently, it is possible to compute for each color point its new coordinates in terms of (a°, b°) chromaticity diagram from its initial coordinates (a*, b*). This computation is based on the set of transformations which correspond to the section of the (a*, b*) chromaticity diagram containing the color point under study.

Our investigation is based on a study of the L*a*b* color space according to the elliptic model developed by MacAdam.[5] Other works have been developed by physiologists to introduce additional visual judgements to the set of experimental parameters in color to improve available data which affect tolerance judgements (Luo and Rigg,[12] McDonald[13] or Berns[14]). However these works have not gathered enough data, and have not yet been enough tested to be used to adequally modeled visual color-differences. Consequently, it seems clever to used the L*a*b* or the L*u*v* color equations than the BFD[12], CMC[13] or ATD[14] equations, to mathematically define a perceptually relevant color metric.

As we have defined it before, our investigation has been developed only in a mathematical point of view. It will be very ingesting to verify our hypotheses according to a physiological point of view and to confirm them from an experimental study. That will be made in forthcoming works.

6. Acknowledgment

For the moment this investigation has been developed in the context of image analysis. As an example consider Figures 18 to 21. The results show that the L°a°b° color space is more appropriated to analyze images, especially when the areas under study are locally homogeneous. Consider for example the blue area for which the visual sermon is finer than for the other areas. We can sets that the results are better h this part for the L°a°b° color space than for the other spaces bee the former is more uniform, more relevant of the chromaticity sensitivities to color differences.

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